

On a Method of Reducing Occultations of Stars by the Moon; together with the Reduction of Occultations observed on Three Occasions at the Liverpool Observatory. By H. C. Plummer, M.A.

The present paper is the result of an undertaking to reduce three series of observations of occultations and to render them available for the discussion of the Moon's semi-diameter. As I was not acquainted with a method which did not seem to leave something to be desired in point of clearness or convenience, I adopted one which offers certain considerable advantages. This method seems to differ essentially from those which have been previously employed, and it therefore appears desirable to describe it in some detail. The first object of the theory is to obtain a quite general equation expressing the geometrical conditions, on the assumption that the figure of the Moon is a sphere and that the line joining the observer to the star is a tangent line to the Moon at the time of the observation.

Let (x, y, z) be the position of the observer; (X, Y, Z) of the centre of the Moon; and (l, m, n) direction cosines of the star, referred to rectangular geocentric axes. Let R be the radius of the Moon. Then

$$\frac{\xi-x}{l} = \frac{\eta-y}{m} = \frac{\zeta-z}{n}$$

$$\frac{\xi-x}{X-x} = \frac{\eta-y}{Y-y} = \frac{\zeta-z}{Z-z}$$

represent the lines joining the observer to the star and to the centre of the Moon respectively. If, now, ψ is the angle between these lines,

$$l(X-x) + m(Y-y) + n(Z-z) = \cos \psi \cdot \sqrt{\{(X-x)^2 + (Y-y)^2 + (Z-z)^2\}};$$

$$R = \sin \psi \cdot \sqrt{\{(X-x)^2 + (Y-y)^2 + (Z-z)^2\}};$$

$$\therefore \{l(X-x) + m(Y-y) + n(Z-z)\}^2 + R^2 = (X-x)^2 + (Y-y)^2 + (Z-z)^2.$$

Now let ρ be the Earth's radius and ϕ the geocentric latitude at the place of observation; T , the sidereal time (or R.A. of the zenith); A, D , the apparent R.A. and declination of the star; α, δ , the geocentric R.A. and declination of the Moon. Also let r, P, s be the Moon's radius vector, parallax, and semi-diameter respectively. Then, if the axis of x has been taken through the First Point of Aries and the axis of z through the Pole,

$$X = r \cos \delta \cos \alpha; \quad Y = r \cos \delta \sin \alpha; \quad Z = r \sin \delta;$$

$$l = \cos D \cos A; \quad m = \cos D \sin A; \quad n = \sin D;$$

$$x = \rho \cos \phi \cos T; \quad y = \rho \cos \phi \sin T; \quad z = \rho \sin \phi.$$

Making these substitutions in the above equation, we obtain

$$R^2 + \{r[\cos \delta \cos D \cos (A - \alpha) + \sin D \sin \delta] - \rho[\cos D \cos \phi \cos (A - T) + \sin D \sin \phi]\}^2 \\ = r^2 + \rho^2 - 2r\rho\{\cos \delta \cos \phi \cos (\alpha - T) + \sin \delta \sin \phi\}.$$

Now

$$\cos \delta \cos D \cos (A - \alpha) + \sin D \sin \delta = \cos \Delta \quad \dots \quad (1)$$

$$\cos D \cos \phi \cos (A - T) + \sin D \sin \phi = \cos Z \quad \dots \quad (2)$$

$$\cos \delta \cos \phi \cos (\alpha - T) + \sin \delta \sin \phi = \cos Z' \quad \dots \quad (3)$$

where Δ is the geocentric distance of the star from the Moon's centre, Z is the true Z.D. of the star and Z' is the true Z.D. of the Moon's centre. Hence

$$R^2 + (r \cos \Delta - \rho \cos Z)^2 = r^2 + \rho^2 - 2r\rho \cos Z'$$

$$\text{or} \quad R^2 = r^2 \sin^2 \Delta + \rho^2 \sin^2 Z - 2r\rho(\cos Z' - \cos \Delta \cos Z).$$

But $r \sin P = 1$; $r \sin s = R$; the Earth's equatorial radius being unity.

Therefore

$$\sin^2 s = \sin^2 \Delta + \rho^2 \sin^2 P \sin^2 Z - 2\rho \sin P (\cos Z' - \cos \Delta \cos Z) \quad (4)$$

and this is the geometrical equation sought.

We are now in a position to derive the equations of condition. Since ρ and P do not occur independently in the fundamental equation, p may be written for $\rho \sin P$. Also put S equal to $\sin^2 s$, and let

$$C = \cos Z' - \cos \Delta \cos Z \quad \dots \quad \dots \quad (5)$$

$$\Sigma = \sin^2 \Delta + p^2 \sin^2 Z - 2pC \quad \dots \quad \dots \quad (6)$$

Then, if the observed and tabular quantities involved were quite accurate, the equation $S = \Sigma$ would be satisfied exactly. Actually, however, the quantities s , p , α , δ , A , D , and t , the mean time of the observation, require corrections ds , dp , da , $d\delta$, dA , dD , and dt . Substituting $s + ds$ for s , &c., an equation is obtained of the form :

$$\Sigma - S = \sin 2s \cdot ds - Bdp - Eda - E'dA - Fd\delta - F'dD - Hdt \quad (7)$$

wherein

$$E = \frac{\partial \Sigma}{\partial \alpha}; \quad F = \frac{\partial \Sigma}{\partial \delta}; \quad E' = \frac{\partial \Sigma}{\partial A}; \quad F' = \frac{\partial \Sigma}{\partial D}; \\ H = \frac{d\Sigma}{dt} = \frac{\partial \Sigma}{\partial T} \cdot \frac{dT}{dt} + \frac{\partial \Sigma}{\partial \alpha} \cdot \frac{d\alpha}{dt} + \frac{\partial \Sigma}{\partial \delta} \cdot \frac{d\delta}{dt} \\ = G + E \frac{da}{dt} + F \frac{d\delta}{dt} \quad \dots \quad \dots \quad \dots \quad (8)$$

and

$$B = \frac{\partial \Sigma}{\partial p} = 2(p \sin^2 Z - C) \dots \dots \dots (9)$$

Hence

$$\begin{aligned} E &= -2 \cos \Delta \frac{\partial}{\partial a} (\cos \Delta) - 2p \left\{ \frac{\partial}{\partial a} (\cos Z) - \cos Z \frac{\partial}{\partial a} (\cos \Delta) \right\} \\ &= -2 \{ (\cos \Delta - p \cos Z) \cos \delta \cos D \sin (A - a) \\ &\quad - p \cos \delta \cos \phi \sin (a - T) \} \dots (10) \end{aligned}$$

$$\begin{aligned} F &= -2 \cos \Delta \frac{\partial}{\partial \delta} (\cos \Delta) - 2p \left\{ \frac{\partial}{\partial \delta} (\cos Z) - \cos Z \frac{\partial}{\partial \delta} (\cos \Delta) \right\} \\ &= -2 \{ (\cos \Delta - p \cos Z) [-\sin \delta \cos D \cos (A - a) + \sin D \cos \delta] \\ &\quad + p [-\sin \delta \cos \phi \cos (a - T) + \cos \delta \sin \phi] \} \dots (11) \end{aligned}$$

and

$$\begin{aligned} G &= \frac{\partial \Sigma}{\partial T} \cdot \frac{dT}{dt} = \frac{dT}{dt} \cdot \left\{ -2p^2 \cos Z \frac{\partial}{\partial T} (\cos Z) \right. \\ &\quad \left. - 2p \left[\frac{\partial}{\partial T} (\cos Z) - \cos \Delta \frac{\partial}{\partial T} (\cos Z) \right] \right\} \\ &= -2p \frac{dT}{dt} \{ (p \cos Z - \cos \Delta) \cos D \cos \phi \sin (A - T) \\ &\quad + \cos \delta \cos \phi \sin (a - T) \} \dots (12) \end{aligned}$$

Finally,

$$\begin{aligned} E' &= -2 \cos \Delta \frac{\partial}{\partial A} (\cos \Delta) + 2p \left\{ \cos \Delta \frac{\partial}{\partial A} (\cos Z) \right. \\ &\quad \left. + \cos Z \frac{\partial}{\partial A} (\cos \Delta) \right\} \\ &= 2 \{ (\cos \Delta - p \cos Z) \cos \delta \cos D \sin (A - a) \\ &\quad - p \cos \Delta \cos D \cos \phi \sin (A - T) \} \dots (13) \end{aligned}$$

and

$$\begin{aligned} F' &= -2 \cos \Delta \frac{\partial}{\partial D} (\cos \Delta) + 2p \left\{ \cos \Delta \frac{\partial}{\partial D} (\cos Z) \right. \\ &\quad \left. + \cos Z \frac{\partial}{\partial D} (\cos \Delta) \right\} \\ &= 2 \{ (\cos \Delta - p \cos Z) [\cos \delta \sin D \cos (A - a) - \cos D \sin \delta] \\ &\quad + p \cos \Delta [-\sin D \cos \phi \cos (A - T) + \cos D \sin \phi] \} \dots (14) \end{aligned}$$

We thus have the means of calculating accurately all the coefficients of the equation of condition. It is only necessary to add, since p is not itself a tabular quantity, that

$$dp = \rho \cos P \cdot dP + \sin P \cdot d\rho \dots \dots (15)$$

It is now necessary to reconsider the above formulæ from the point of view of the computer. In the first place $\cos Z$ and $\cos Z'$ can be found by means of (2) and (3). It is assumed that 7-figure logarithms are used, and it may be pointed out that perfect accuracy will be attained if the terms of Σ , none of which in fact exceeds .0005, are calculated to ten places of decimals. Equation (1) is unsuitable for obtaining $\sin^2 \Delta$, but from it can be derived

$$\begin{aligned}\sin^2 \Delta &= 1 - \cos^2 \Delta = 1 - \{\cos(D - \delta) - 2 \cos \delta \cos D \sin^2 \frac{1}{2}(A - \alpha)\}^2 \\ &= \sin^2(\delta - D) + 4 \cos \delta \cos D \cos(\delta - D) \sin^2 \frac{1}{2}(A - \alpha) \\ &\quad - 4 \cos^2 \delta \cos^2 D \sin^4 \frac{1}{2}(A - \alpha) \\ &= \sin^2(\delta - D) + \cos D \cos \delta \cos(\delta - D) \sin^2(A - \alpha) \\ &\quad + \sin 2D \sin 2\delta \sin^4 \frac{1}{2}(A - \alpha) \quad \dots \quad \dots \quad \dots \quad (16)\end{aligned}$$

In this form the last term is generally negligible, but in any case it is easily added as a small correction; its importance depends only on the fact that it is always positive. From $\sin^2 \Delta$ can be deduced $\cos \Delta$, and then we have C from (5). This is the most convenient way of calculating C , but it must be said that the resulting value of $2pC$ cannot be trustworthy beyond nine places, since C is known only to seven places of decimals. This falls short of the highest standard of accuracy, but I have generally contented myself with this method: partly because of its convenience, partly because the error introduced is an *accidental* one, small in comparison with those which depend on the star's place and the observed time, and partly because other methods failed to reveal errors in the simpler process, carefully performed, such as seemed to necessitate or warrant the extra labour involved by their adoption. Two alternatives may, however, be mentioned here. The angles Z and Z' are known from their cosines. Then (5) may be written

$$C = 2 \sin \frac{1}{2}(Z + Z') \sin \frac{1}{2}(Z - Z') + 2 \cos Z \sin^2 \frac{1}{2} \Delta \quad \dots \quad (17)$$

Or referring to the *geocentric* spherical triangle formed by the zenith (Z), the Moon's centre (M) and the star (S), it is seen that

$$C = \cos Z' - \cos \Delta \cos Z = \sin \Delta \sin Z \cos ZSM.$$

But if P is the pole, $ZSM = PSM + ZSP = \varpi + \zeta$, say.

$$\begin{aligned}\therefore C &= \sin \Delta \sin Z \cos(\varpi + \zeta) \\ \text{where } \sin \varpi &= \cos \delta \sin(A - \alpha) / \sin \Delta \\ \sin \zeta &= \cos \phi \sin(T - A) / \sin Z\end{aligned} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \dots \quad \dots \quad (18)$$

These latter equations may not give the auxiliary angles without ambiguity: in that case reference to a figure will remove the difficulty. The use of equations (18) involves considerable extra labour, but they possess the advantage that a comparison of the

value of C so obtained with the value obtained directly from (5) furnishes a very complete if not a very accurate check on the work. There is no further difficulty in finding $\Sigma - S$, which is at present expressed in circular measure.

In finding the coefficients of equation (7) 5-figure logarithms are of course amply sufficient. In my own computations, B has been found from equation (9). In finding E , F and G the following slightly modified approximate formulæ have been used :—

$$E = -2 \cos \delta \{ (1 - p \cos Z) \cos D \sin (A - \alpha) - p \cos \phi \sin (a - T) \} \quad \dots \quad (19)$$

$$F = -2 \{ (1 - p \cos Z) \sin (D - \delta) + p [\sin \phi \cos \delta - \sin \delta \cos \phi \cos (T - \alpha)] \} \quad \dots \quad (20)$$

$$G = 2p \frac{dT}{dt} \cos \phi \cos D \{ \sin (A - \alpha) \cos (A - T) - p \cos Z \sin (A - T) \} \quad \dots \quad (21)$$

In equation (8) $\frac{da}{dt}$ and $\frac{d\delta}{dt}$ are most easily expressed in seconds of arc per second of mean time; for the sake of uniformity therefore we take $\log \left(2 \frac{dT}{dt} \right) = 1.47831$. It is easy to see, either by comparing equations (10) and (13), and (11) and (14), or from general considerations, that $E' = -E$ and $F' = -F$ approximately. In consequence of this it has not been thought worth while to calculate the coefficients E' and F' separately. In fact these coefficients are not needed with great accuracy, for if considerable values be assigned to dA and dD , terms of the second order will become necessary in equation (7). The term in $d\rho$ is most convenient if we are dealing with observations made at a single observatory. If we have to combine observations made at different observatories, it will be necessary to remove the term in $d\rho$ in equation (15) and to use $-B\rho \cos P$ (which differs little from $-B$) as the coefficient of a term in dP on the right-hand side of equation (7). If it is desired to improve ρ by adding $d\rho$, the quantity $B \sin P d\rho$ must be added to the left-hand side of the same equation. For an increase of altitude of 209 feet, $d\rho = .00001$, while $B \sin P$ is not likely to exceed .0001. In the final equation we divide $\Sigma - S$ by $\sin 1''$ in order to transform the residual from circular measure into seconds of arc, and divide the whole equation by $\sin 2s$ so as to make the coefficient of ds unity.

It is well known that refraction exercises an effect, which is of a parallactic nature, on the circumstances of an occultation. If there were no terrestrial atmosphere it would be necessary for the observer to change his position in order to see the occultation at the same time that he makes his observation under actual conditions. The effect, which is generally evanescent and always small, is presumably neglected in the Greenwich reductions, since

no reference is made to it, but it is taken into account by those who follow Bessel, as for instance by Küstner in his admirable memoir on "Nine Occultations of the *Pleiades*" It is easy to see that the effect is allowed for, if the radius ρ is increased by an amount x such that

$$1+x=\mu \sin z/\sin Z$$

where μ is the refractive index of the air at the surface of the Earth, and z is the apparent Z.D. of the star (cf. *Chauvenet's Astronomy*, 5th ed., vol. i., p. 516). Now $z=Z-a \tan Z$ where $a \tan Z$ is small. Consequently

$$1+x=\mu(\sin Z-a \sin Z)/\sin Z=\mu(1-a)$$

or

$$x=\mu-1-\mu a=\mu-1-a.$$

It is quite clear that an accurate calculation of this quantity can be made only if μ and a are known with great exactness. If we adopt Gladstone and Dale's law as to the dependence of the refractive index on pressure, and Mascart's numbers (*Comptes Rendus*, t. lxxviii., p. 617) for the index itself and the temperature coefficient, we have

$$\mu-1=0.002923(1-0.00383t)\frac{p}{p_0}.$$

And using Bessel's formula and numbers for the refraction we have

$$a=a\beta^A\gamma^\lambda=a\left(\frac{p}{p_0}\right)^{1+m}(1-\epsilon t)^{1+n}$$

where m and n are small, and $\epsilon=0.00364$. At the zenith, when $t=0^\circ\text{C}$ and $p=760$ mm., $a=0.002927$, we may put then $a=0.002927-a'$ for any Z.D., and let $p/p_0=1+q$. If we make these transformations in the expression for x , we obtain a formula which is certainly erroneous in a slight degree, inasmuch as it gives a negative value of x at the zenith when $q=t=0$. This may be due to the fact that Bessel's temperature coefficient 0.00364 is too small. However it is only necessary to retain terms amounting to 0.000001 , and from an examination in detail it is concluded that we may take $x=a'(1+q-0.00364t)$, and that no serious error will be introduced by assuming, as has been generally done, a mean state of the atmosphere, in which case we find the following:—

Z=	0°	50°	60°	65°	70°	75°	80°	85°
x=	0.000 000	000	001	002	003	005	012	038

It is to be feared that the constants of refraction may be insufficiently exact in the only case where they are of importance here, i.e. at great zenith distances. For this reason the correction has not been applied in the reductions which follow, but its

magnitude in the few instances in which it amounts to $0''.01$ may be recorded here, though, as a matter of fact, in no case has it any importance. In the final equations the coefficient of $dp = -B/\sin 2s = B'$ say, and the residual is $(\Sigma - S)/\sin 2s \sin 1''$. It is only necessary then to add $B \sin P \cdot x/\sin 2s \sin 1'' = -B'Px$, where P is expressed in seconds, to correct the final equation for the effect of refraction. The equations affected are confined to Series I, and the corrections are $-0''.02$ in No. 3, $-0''.01$ in Nos. 5, 6, 7, and 9, and $+0''.01$ in No. 11.

The accompanying reductions refer to observations made by the Director of the Liverpool Observatory on the following dates :—

1897 July 23 (*Monthly Notices*, vol. lviii., p. 20)

1898 Dec. 27 („ „ lix., p. 165)

1899 Dec. 16 (,, ,, lx., p. 222)

On the first of these dates the *Pleiades* were occulted, and on the other two the Moon was eclipsed. A few corrections have been made in the designations of the stars, and in several instances it has not been found possible to identify the stars occulted. In Series I the mean places of the stars have been taken from Elkin's memoir on the *Pleiades*, except in those cases where the apparent places were given in the *Nautical Almanac* of the year. In columns 5 and 6 are to be found apparent coordinates (A and D) as used in the reduction. The stars in Series II are all taken directly from a list prepared by the photographic method at the Poulkova Observatory (*Astr. Nachr.*, No. 3533). For Series III the three stars B.D. $+22^{\circ}$, Nos. 999, 1004 and 1006, were taken from the *Astr. Ges.* Catalogue (Berlin). The coordinates of No. 1011 are the result of my own reduction of a plate taken at the University Observatory, Oxford, by Mr. Bellamy, to whom, as well as to Professor Turner, I am indebted for kind assistance in this connection. The corrections deduced from the plate to the Berlin meridian places are these :—

B.D. + 22, 999	$\Delta A = -0.05^s$	$\Delta D = -0.4''$
1004	+ 0.02	- 0.2
1006	+ 0.02	+ 0.2

As regards the places of the Moon (α and δ), columns 7 and 8 contain the values deduced by interpolation from the *Nautical Almanac* with the following assumed corrections added :—

	^s	
1897 July 23	$\Delta\alpha = +0.06$	$\Delta\delta = -2''.0$
1898 Dec. 27	$+0.20$	$+2.3$
1899 Dec. 16	$+0.38$	$+0.8$

These are based on observations made at Greenwich, in the third case directly, and in the two former by estimation, not very reliable probably, from the results of previous and subsequent nights. The unpublished results were kindly communicated to me by the Astronomer Royal. The assumed position of the observatory is that given in the *Nautical Almanac*, and no correction has been made for the altitude, which is about '000011 in units of the Earth's radius. The omission contributes to the term dp , and might be allowed for in the way already explained. But this is unnecessary if the term is eliminated from the final equations, and if this is done the result will be independent of the assumed figure of the Earth.

I. Occultation of the Pleiades, 1897 July 23.

Refer- ence No.	Name of Star.	Phase.	Sidereal Time.		Star's R.A.		Star's Decl.		Moon's R.A.		Moon's Decl.		Moon's Parallax.	Moon's Semi- diameter.
			h	m	s	h	m	s	h	m	s	°	'	''
1	Electra	D.	20	17	23.16	3	38	47.72	23	47	32.1	24	22	54.3
2	8	"	20	52	17.98	3	40	8.60	3	36	55.60	24	26	22.0
3	Merope	"	20	52	56.59	3	40	14.96	3	36	56.96	24	26	25.8
4	10	"	21	1	50.90	3	40	21.90	3	37	15.83	24	27	18.4
5	18	"	21	16	45.59	3	41	14.14	3	37	47.43	24	28	46.2
6	p	"	21	17	31.11	3	41	15.21	3	37	49.04	24	28	50.7
7	Aleyone	"	21	21	28.16	3	41	23.92	3	37	57.42	24	29	13.8
8	29	"	21	54	28.77	3	42	24.22	24	1	55.0	24	32	26.6
9	Pleione	"	22	13	9.53	3	43	5.70	3	39	47.08	24	34	14.5
10	32	"	22	17	48.19	3	43	15.64	3	39	56.94	24	34	41.3
11	9	R.	21	47	45.87	3	40	10.84	3	38	53.18	24	31	47.4
12	Aleyone	"	22	15	46.66	3	41	23.92	3	39	52.64	24	34	29.6

II. Occultations during the Lunar Eclipse, 1898 December 27.

Refer- ence No.	Name of Star.	Phase.	Sidereal Time.		Star's R.A.		Star's Decl.		Moon's R.A.		Moon's Decl.		Moon's Parallax.	Moon's Semi- diameter.
			h	m	s	h	m	s	h	m	s	°	'	''
1	R.D. + 23 1392	D.	4	59	30.11	6	26	53.84	23	20	35.7	23	34	28.4
2	P.D. + 23 1402	"	5	3	32.66	6	27	42.12	6	25	47.17	23	34	12.0
3	B.D. + 23 1403	"	5	13	6.43	6	27	57.08	6	26	7.61	23	33	33.2

Refer- ence No.	Name of Star.	Phase.	Sidereal Time.			Star's R.A.			Star's Decl.			Moon's R.A.			Moon's Decl.			Moon's Parallax.	Moon's Semi- diameter.	
			h	m	s	h	m	s	h	m	s	h	m	s	h	m	s	'	''	
4	B.D.+22 1385	D.	5	37	32.64	6	28	4.28	22	51	36.1	6	26	59.90	23	31	53.0	54	6.54	14 46.12
5	B.D.+23 1407	"	5	58	24.96	6	28	31.15	23	16	40.0	6	27	44.53	23	30	26.5	54	6.41	14 46.08
6	B.D.+22 1392	"	6	0	7.44	6	29	3.42	22	56	19.6	6	27	48.18	23	30	19.4	54	6.40	14 46.08
7	B.D.+23 1415	"	6	26	55.14	6	29	51.71	23	1	56.0	6	28	45.44	23	28	27.3	54	6.23	14 46.04
8	B.D.+23 1416	"	6	27	40.36	6	29	53.23	23	0	58.3	6	28	47.05	23	28	24.1	54	6.22	14 46.04
9	B.D.+22 1369	R.	5	26	21.10	6	26	27.11	22	54	31.8	6	26	35.97	23	32	39.0	54	6.61	14 46.14
10	B.D.+23 1389	"	5	39	13.62	6	26	35.70	23	11	28.6	6	27	3.50	23	31	46.1	54	6.53	14 46.12
11	B.D.+23 1407	"	6	20	9.13	6	28	31.15	23	16	40.0	6	28	30.98	23	28	55.7	54	6.27	14 46.05
12	B.D.+23 1402	"	6	23	12.12	6	27	42.12	23	7	57.0	6	28	37.50	23	28	42.9	54	6.25	14 46.04
13	B.D.+22 1385	"	6	32	30.15	6	28	4.28	22	51	36.1	6	28	57.37	23	28	3.7	54	6.19	14 46.03
14	B.D.+23 1403	"	6	32	40.18	6	27	57.08	23	7	29.3	6	28	57.73	23	28	3.0	54	6.19	14 46.03

III. Occultations during the Lunar Eclipse, 1899 December 16.

Refer- ence No.	Name of Star.	Phase.	Sidereal Time.			Star's R.A.			Star's Decl.			Moon's R.A.			Moon's Decl.			Moon's Parallax.	Moon's Semi- diameter.
			h	m	s	h	m	s	°	'	''	h	m	s	°	'	''		
1	B.D. + 22 1004	D.	6	27	28.95	5	37	1.77	22	37	38.0	5	36	43.27	22	56	30.5	56'	15 20.18
2	B.D. + 22 1006	"	6	36	36.24	5	37	9.10	22	39	8.3	5	37	4.39	22	56	15.4	56'	15 20.12
3	B.D. + 22 1011	"	6	44	25.03	5	37	45.09	22	30	33.2	5	37	22.47	22	56	2.3	56'	15 20.08
4	B.D. + 22 999	R.	6	58	22.01	5	36	12.17	22	32	54.08	5	37	54.74	22	55	38.6	56'	15 19.99
5	B.D. + 22 1006	"	7	18	20.79	5	37	9.10	22	39	8.3	5	38	40.95	22	55	3.9	56'	15 19.88

I. *Occultation of the Pleiades, 1897 July 23.*

1.	$+1^{\text{h}}25 = ds$	$-0.0241 dp$	$+0.7364 da$	$+0.5833 d\delta$	$+0.6497 dt$
2.	$+4^{\text{h}}17 = ds$	$+0.0248 dp$	$+0.7511 da$	$+0.5680 d\delta$	$+0.6403 dt$
3.	$+2^{\text{h}}02 = ds$	$+0.8683 dp$	$+0.8256 da$	$-0.4379 d\delta$	$+0.5811 dt$
4.	$+2^{\text{h}}06 = ds$	$-0.1375 dp$	$+0.6379 da$	$+0.7121 d\delta$	$+0.5861 dt$
5.	$+7^{\text{h}}11 = ds$	$+0.5206 dp$	$+0.9125 da$	$+0.1109 d\delta$	$+0.6690 dt$
6.	$-3^{\text{h}}33 = ds$	$+0.5895 dp$	$+0.9047 da$	$-0.0007 d\delta$	$+0.6520 dt$
7.	$-2^{\text{h}}03 = ds$	$+0.6566 dp$	$+0.9065 da$	$-0.0739 d\delta$	$+0.6439 dt$
8.	$+5^{\text{h}}14 = ds$	$+0.1032 dp$	$+0.7607 da$	$+0.5634 d\delta$	$+0.6043 dt$
9.	$+0^{\text{h}}45 = ds$	$+0.8544 dp$	$+0.8055 da$	$-0.4760 d\delta$	$+0.5115 dt$
10.	$+3^{\text{h}}65 = ds$	$+0.2025 dp$	$+0.8065 da$	$+0.4623 d\delta$	$+0.5996 dt$
11.	$-5^{\text{h}}08 = ds$	$-0.5788 dp$	$-0.9041 da$	$-0.0263 d\delta$	$-0.3391 dt$
12.	$-4^{\text{h}}40 = ds$	$-0.0049 dp$	$-0.6912 da$	$-0.6449 d\delta$	$-0.2701 dt$

II. *Occultations during the Lunar Eclipse, 1898 December 27.*

1.	$+5^{\text{h}}63 = ds$	$-0.4064 dp$	$+0.3122 da$	$+0.9433 d\delta$	$+0.0961 dt$
2.	$+2^{\text{h}}85 = ds$	$+0.1580 dp$	$+0.9140 da$	$+0.1099 d\delta$	$+0.3881 dt$
3.	$+2^{\text{h}}20 = ds$	$+0.1345 dp$	$+0.9130 da$	$+0.1113 d\delta$	$+0.3814 dt$
4.	$+4^{\text{h}}95 = ds$	$+0.5078 dp$	$+0.4789 da$	$-0.8603 d\delta$	$+0.2416 dt$
5.	$-0^{\text{h}}80 = ds$	$-0.4159 dp$	$+0.3996 da$	$+0.8985 d\delta$	$+0.1036 dt$
6.	$+0^{\text{h}}80 = ds$	$+0.2968 dp$	$+0.8192 da$	$-0.4530 d\delta$	$+0.3574 dt$
7.	$+1^{\text{h}}64 = ds$	$-0.0129 dp$	$+0.9181 da$	$+0.0419 d\delta$	$+0.3480 dt$
8.	$+5^{\text{h}}96 = ds$	$+0.0157 dp$	$+0.9232 da$	$-0.0189 d\delta$	$+0.3538 dt$
9.	$-1^{\text{h}}21 = ds$	$+0.2531 dp$	$-0.6465 da$	$-0.7074 d\delta$	$-0.4214 dt$
10.	$-0^{\text{h}}14 = ds$	$-0.3473 dp$	$-0.8073 da$	$+0.4738 d\delta$	$-0.3222 dt$
11.	$-2^{\text{h}}38 = ds$	$-0.4797 dp$	$-0.0493 da$	$+0.9946 d\delta$	$-0.0810 dt$
12.	$-2^{\text{h}}32 = ds$	$-0.2240 dp$	$-0.8311 da$	$+0.4265 d\delta$	$-0.3449 dt$
13.	$-0^{\text{h}}52 = ds$	$+0.3234 dp$	$-0.7184 da$	$-0.6216 d\delta$	$-0.2316 dt$
14.	$+3^{\text{h}}43 = ds$	$-0.2108 dp$	$-0.8273 da$	$+0.4402 d\delta$	$-0.3478 dt$

III. *Occultations during the Lunar Eclipse, 1899 December 16.*

1.	$-3^{\text{h}}24 = ds$	$-0.4330 dp$	$+0.6954 da$	$+0.6501 d\delta$	$+0.2544 dt$
2.	$-2^{\text{h}}77 = ds$	$-0.4950 dp$	$+0.5822 da$	$+0.7711 d\delta$	$+0.1988 dt$
3.	$-3^{\text{h}}59 = ds$	$-0.2908 dp$	$+0.8910 da$	$+0.2406 d\delta$	$+0.3431 dt$
4.	$+0^{\text{h}}26 = ds$	$-0.0643 dp$	$-0.7082 da$	$+0.4364 d\delta$	$-0.4354 dt$
5.	$+2^{\text{h}}52 = ds$	$-0.1817 dp$	$-0.4068 da$	$+0.8982 d\delta$	$-0.2492 dt$

It was obvious, of course, that the above reductions were altogether insufficient in number to give by themselves any reliable value of the semi-diameter of the Moon, but an attempt was made to examine in a general manner the results which

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might be deduced from the equations. The attempt was not successful, and it is not without interest to consider why this was so. The case of the ten disappearances of stars in the *Pleiades*, I. 1-10, is typical. The equations were given equal weights after division by the coefficients of dt , which term was afterwards neglected of course. The star places are known to be very accurate, and the residual, therefore, is fairly attributable to the error in the observed quantity—the time, and can be treated in accordance with the theory of least squares. Theoretically, this method seems reasonable, but it may have the result of giving an overwhelming weight to the result of an occultation at a point of elevation or depression in the Moon's figure. It would seem, then, that a compromise is desirable in some cases, but the question is one which I do not pretend to decide. As regards the equations referred to, this difficulty did not exist, and the four normal equations were at once formed in ds , dp , da and $d\delta$. It was then found that these equations were not, in any real sense, consistent and independent, as would be necessary for the determination of the four unknowns. In fact, they could be reduced to two without sacrificing any of the information which they were capable of giving. This result can be referred, I think, to two simple causes. In the first place, the observations do not extend over a long interval of time, and the Moon's change of position is small. Now in any position it is clear that if a compensating change were made in the radius of the Moon, the Moon's distance from the observer might be changed without any effect on the phenomena. Consequently the Moon's *apparent* semi-diameter and parallax cannot be determined independently from occultations observed during a short time at a single observatory. In a similar way if disappearances alone are considered, it is evident that the effect of moving the centre of the Moon along its own apparent path while changing at the same time its semi-diameter so that the advancing point of the edge retains its position, will be very small in the case of occulted stars which lie near that path. In fact, in both cases we shall be trying to determine small quantities of the second order, and it is inevitable that the results will be expressed in terms of the residual errors. The difficulty can only be overcome by combining results obtained at distant observatories, and it is in the hope that the reductions given above may be utilised in this manner that this work has been undertaken and published.

Oxford: 1901 January 7.

Observations of Occultations of Stars and Saturn by the Moon, made at the Royal Observatory, Greenwich,
in the Year 1900.

(Communicated by the Astronomer-Royal.)

Day.	Phenomenon.	Telescope.	Power.	Moon's Limb.	Mean Solar Time of Observation.	Observer.
1900.					h m s	
Jan. 9	Disapp. 27 Arietis	Sheepshanks Equat.	55	Dark	9 15 23.8	AC.
9 (a)	"	28-inch Equat.	670	"	9 15 23.8	B.
Feb. 6	" 8 Arietis	"	670	"	8 35 20.94	B.
6	"	Sheepshanks Equat.	55	"	8 35 20.71	WB.
6	"	Corbett Equat.	100	"	8 35 20.34	HF.
6	"	Old Altazimuth	100	"	8 35 21.05	S.
6	Reapp. "	"	100	Bright	9 25 29.35	S.
7 (b)	Disapp. A ² Tauri	Sheepshanks Equat.	55	Dark	7 2 25.77	R.
April 4 (c)	" o Tauri	Astrographic Equat.	225	"	9 29 18.84	H.
4	"	Sheepshanks Equat.	55	"	9 29 19.86	R.
May 7	" 19 Sextantis	Old Altazimuth	100	"	10 42 2.20	S.
15	Reapp. 15 Ophiuchi	Sheepshanks Equat.	100	"	13 58 22.55	B.
June 2	Disapp. κ Caneri	Old Altazimuth	100	"	8 31 35.36	H.
July 11	" ζ ² Sagittarii	Sheepshanks Equat.	55	"	12 18 44.25	AC.
11	"	Astrographic Equat.	225	"	12 18 44.24	S.